PARKFIELD PRIMARY SCHOOL

CALCULATIONS POLICY

INTRODUCTION

This policy outlines the school's agreed progression through written strategies for addition, subtraction, multiplication and division.

Children work through the progression so that they can understand, use, apply and explain a compact method of calculation by the end of Year 6, although it is expected that children throughout the school will be able to explain and apply the current mental and written methods they are using.

As children will move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However we would expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum.

The policy also includes examples and diagrams, showing how we teach calculations, as consistency in layout and presentation is important to support learning and progression. The policy also includes the equipment and resources that will be used to support children's understanding of the strategy.

The importance of mental maths

While this policy focuses on written calculations in maths, we recognise the importance of the mental strategies and known facts that form the basis of all calculations. The following checklists outline the key skills and number facts that children are expected to develop throughout the school.
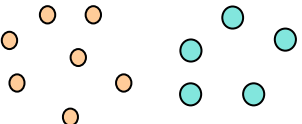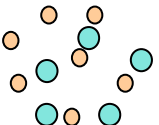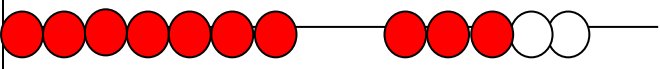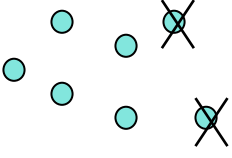
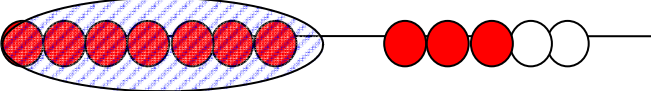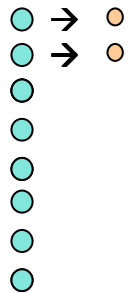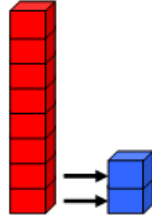To add and subtract successfully, children should be able to:
- Recall all addition pairs to 9 + 9 and number bonds to 10
- Recognise addition and subtraction as inverse operations
- Add mentally a series of one digit numbers *(e.g. 5 + 8 + 4)*
- Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value *(e.g. 600 + 700, 160 — 70)*
- Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways *(e.g. partition 74 into 70 + 4 or 60 + 14)*

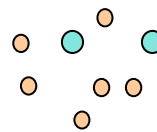To multiply and divide successfully, children should be able to:
- Recall all multiplication facts to 12 x 12
- Calculate products such as 50 x 7, 50 x 70, 700 x 5 or 700 x 50 using the related fact 7 x 5 and their knowledge of place value, including patterns of multiplying and dividing by the powers of 10
- Add 2 or more single digit numbers mentally
- Partition 2 and 3 digit numbers into multiple of 100, 10 and 1 in different ways
- Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value
- Recognise multiplication and division as inverse operations
- Know subtraction facts to 20 and use this knowledge to subtract multiples of 10 *(e.g. 120—80, 320—90)*
- Use tables knowledge to estimate how many times one number divided into another *(e.g. how many 6s there are in 47 or how many 23s there are in 92)*

Progression in Addition and Subtraction

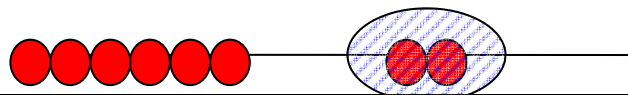| ADDITION	SUBTRACTION
Combining two sets (aggregation model)	**Taking away (separation model)**
Putting together – two or more amounts or numbers are put together to make a total.	Where one quantity is taken away from another to calculate what is left.
Using a wide range of objects; including objects of mixed sets and Base 10 ones.	$8 - 2 = 6$
Count one, then the other. Combine the sets and count again. Starting at 1.	Multilink towers- to physically take away objects.
Counting along the bead bar, count out the 2 sets and then draw them together.	
Combining two sets (augmentation model)	**Finding the difference (comparison model)**
Increasing – when one amount is made bigger Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.	Two quantities are compared to find the difference. $8 - 2 = 6$
Counters- 7 then count on 8, 9, 10, 11, 12.	
	Cuisenaire Rods
Number tracks- start on 7 and then count on 5 more jumps.	
	Multilink towers
Bead strings- make a set of 7 and a set of 5. Then count on from 7.	

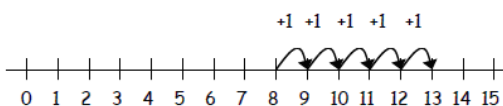|	**1 set within another (part-whole model)**
	The quantity in the whole set and one part are known, and may be used to find out how many are in the unknown part.
	Bead strings
	8 – 2 = 6

Bridging through 10 U+U, TU+U
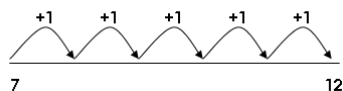
This method teaches children to become more efficient.

Labelled number line

8 + 5 = 13

Blank number line
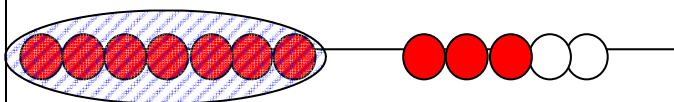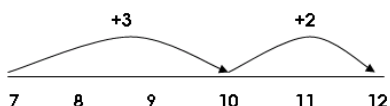
7 + 5 = 12

7 + 5 is broken down into 7 + 3 + 2 using questions such as 'How many more to the next multiple of 10?' and then 'If we use 3 of the 5 to get to 10, how many more do we need to add on?'

This encourages children to use and apply their knowledge of number bonds to 10.
Bead strings are used to illustrate addition including bridging through a tens boundary by counting on 3 and then 2 more.

Steps in addition can be recorded on a number track and number line. The steps often bridge through a multiple of 10.
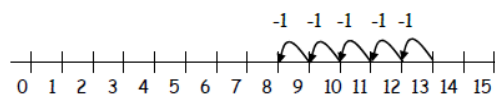
Bridging through 10 TU-U
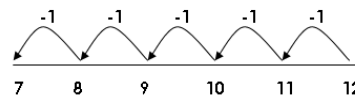
This method teaches children to become more efficient.
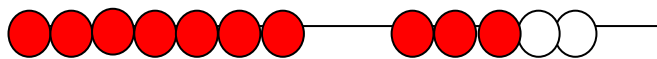
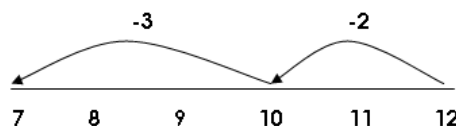Labelled number line

13 – 5 = 8

Blank number line

12 – 5 = 7

12 – 5 is broken down into 12 – 2 – 3 using questions such as 'How many more to the last t multiple of 10?' and then 'If we use 2 of the 5 to get to 10, how many more do we need to subtract?'

This encourages children to use and apply their knowledge of number bonds to 10.
Bead strings are used to illustrate subtraction including bridging through a tens boundary by counting back 2 and then 3 less.
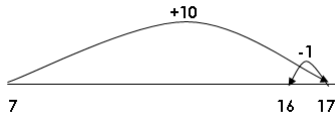
Steps in subtraction can be recorded on a number track and number line. The steps often bridge through a multiple of 10.

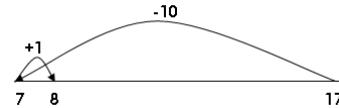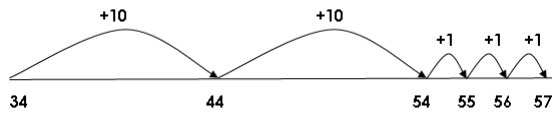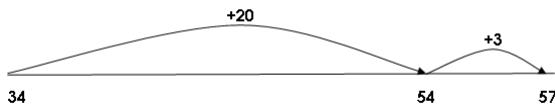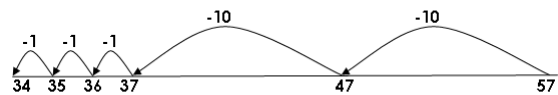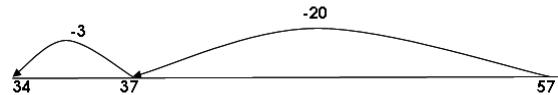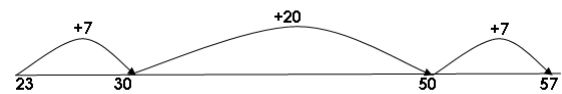|	
Compensation method TU+U, TU+TU Children are also taught this method when they are adding numbers close to a multiple of 10, such as 9, 11, 19, 21, etc. For example, 7 + 9 would be treated as 7+10 and then adjusted by subtracting 1. 	**Compensation method** Children are also taught this method when they are subtracting numbers close to a multiple of 10, such as 9, 11, 19, 21, etc. 17 – 9 is treated as 17 – 10 and then adjusted by adding 1.
Working with larger numbers- partitioning TU+TU Children will continue to use empty number lines, starting with the larger number and counting on. They may use compensation where appropriate. 34 + 23 = 57 When comfortable, children will be encouraged to combine the tens into a single jump and then the ones into another jump (or 2 jumps if crossing a tens boundary). 34 + 23 = 57 To prepare children for working with Base 10 equipment and for bridging through tens or hundreds boundaries, we then ensure children always start working with the ones. All of these strategies continue to be supported through using bead strings and Numicon. Children should also be introduced to columnar recording of calculations. 	**Working with larger numbers- partitioning TU-TU** Children will continue to use empty number lines, starting with the larger number and counting on. They may use compensation where appropriate. 57 – 23 = 34 57 – 23 = 34 The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 23 to 57 in steps totalling 34. All of these strategies continue to be supported through using bead strings and Numicon. Children should also be introduced to columnar recording of calculations.

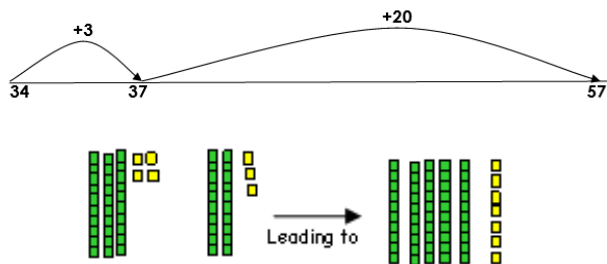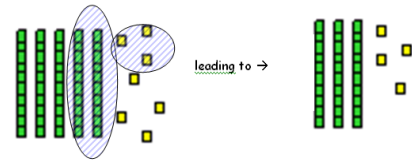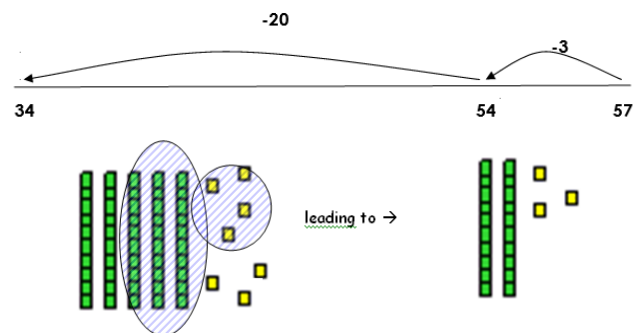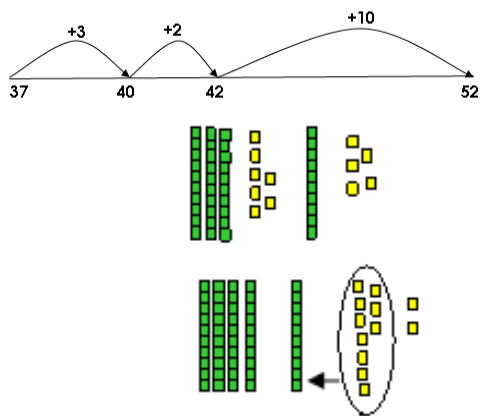| **Aggregation model of addition**	**Separation model of subtraction**
Children combine 2 sets, ones with ones and tens with tens, starting with 1 (or 10). We need to ensure that children always start working with the ones. 34 + 23 = 57 When children are confident with this model of addition, we move them back to the augmentation model.	Children combine 2 sets, ones with ones and tens with tens, starting with 1 (or 10). We need to ensure that children always start working with the ones. 57 – 23 = 34 leading to →
Augmentation model of addition	**Comparison and part-whole models of subtraction**
Encourage children to begin to count out from the first set of ones and tens, avoiding the need to start at 1 (or 10). +3 +20 34 37 57 Leading to At this stage, children can begin to use an informal method to support, record and explain their methods. 4 + 30 + 3 + 20 = 7 50 57	Finding the difference, or counting on. Find the difference between 57 and 34. -20 -3 34 54 57 leading to → At this stage, children can begin to use an informal method to support, record and explain their methods. 50 + 7 – 30 + 4 = 20 3 23

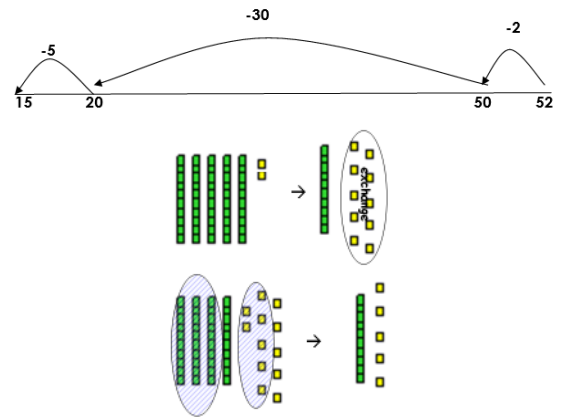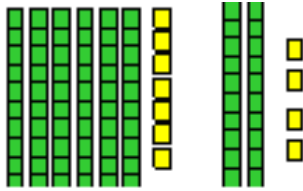|	
Bridging through 10 with larger numbers. Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a tens rod, which is equivalent to crossing the tens boundary on the number line. We encourage children to adopt the same approach of getting to the next multiple of 10 first, then adding the rest of the ones, and finally adding the tens. This allows children to use their knowledge of number bonds. 37 + 15 = 52 	**Bridging through 10 with larger numbers** Once secure in partitioning for subtraction, children return to exchanging. What happens if there are not enough ones? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a tens rod, which is equivalent to crossing the tens boundary on the number line. We encourage children to adopt the same approach of getting to the previous multiple of 10 first, then subtracting the rest of the ones, and finally subtracting the tens. This allows children to apply their knowledge of number bonds. 52 – 37 = 15

Expanded vertical method

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning recording and the expanded vertical method.

67 + 24 = 91


```
60  +  7  +  20  +  4 =
        80        11
            91
```

This layout shows the addition of the tens to the tens and the ones to the ones separately. To find the partial calculations children add the ones first, and the total of the partial calculations can be found by recombining the sums of the tens and ones.

The addition of the tens in the calculation 67 + 24 is described in the words 'sixty plus twenty equals eighty', stressing the link to the related fact 'six plus two equals eight'.

```
    67        60 + 7
+   24        20 + 4
    11         7 + 4
    80        60 + 20
    91
```

Expanded vertical method

91 – 67 = 24


```
90  +  1  -  60  +  7 =
80  +  11  -  60  +  7 =
        20        4
            24
```

Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.

```
    80        11
    90    +    1
-   20    +    4
    60    +    7
```

Compact method

The expanded method leads children to the more compact method so that they understand its structure and efficiency.

In this method, recording is reduced further. 'Carry' digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.
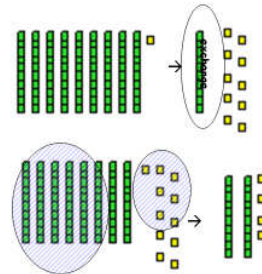
Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

```
    67
+   24
    91
     1
```

Compact method (decomposition)

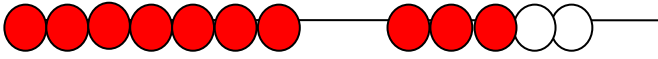The expanded method leads children to the more compact method so that they understand its structure and efficiency.

Children are able to use the compact method, crossing tens and hundreds boundaries and dealing with zeros.

```
    80 11
    91
-   67
    24
```

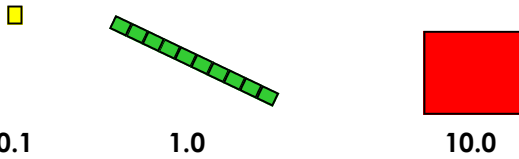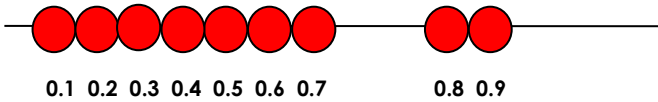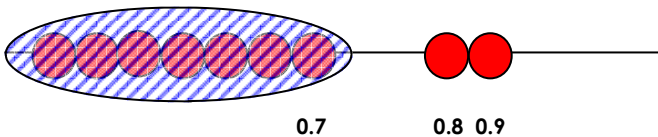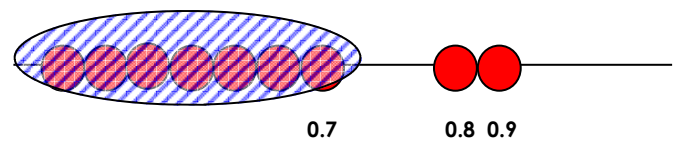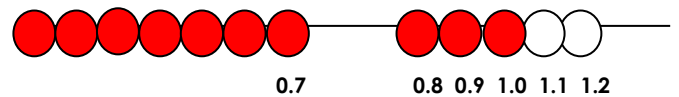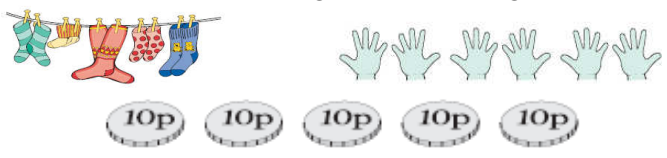| Addition of decimals	Subtraction of decimals
Addition of decimals We return to the models that the children are familiar with. Ensure that the children are confident in counting decimals. Counting: lots of practicing backwards and forwards, etc. Bead strings: each bead representing 0.1 Base 10 equipment: each different block of colour is 1.0 **0.1** **1.0** **10.0** Aggregation model of addition: counting both sets– starting at zero. 0.7 + 0.2 = 0.9 **0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9** Augmentation model of addition: starting from the first set total, count on to the end of the second set. 0.7 + 0.2 = 0.9 **0.7 0.8 0.9** Bridging through 1.0: encouraging connections with number bonds. 0.7 + 0.5 = 1.2 **0.7 0.8 0.9 1.0 1.1 1.2** Partitioning 3.7 + 1.5 = 5.2 	**Subtraction of decimals** Separation model of subtraction 0.9 - 0.7 = 0.2 **0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9** Comparison model of subtraction 0.9 - 0.7 = 0.2 **0.7 0.8 0.9** Bridging through 1.0: encouraging connections with number bonds. 1.2 - 0.5 = 0.7 **0.7 0.8 0.9 1.0 1.1 1.2** Partitioning 5.2 - 1.5 = 3.7

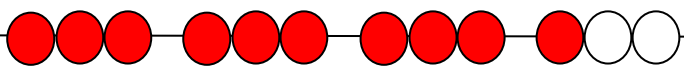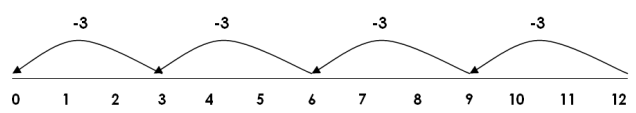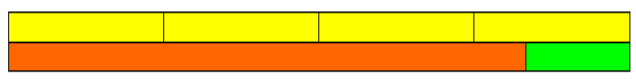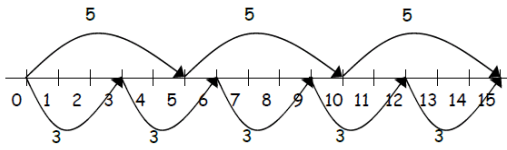| Gradation of difficulty- addition	Gradation of difficulty- subtraction
1. No exchange 2. Extra digit in the answer 3. Exchanging ones to tens 4. Exchanging tens to hundreds 5. Exchanging ones to tens and tens to hundreds 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Add two or more decimals with a range of decimal places.	1. No exchange 2. Fewer digits in the answer 3. Exchanging tens for ones 4. Exchanging hundreds for tens 5. Exchanging hundreds to tens and tens to ones 6. More than two numbers in calculation 7. As 6 but with different number of digits 8. Decimals up to 2 decimal places (same number of decimal places) 9. Subtract two or more decimals with a range of decimal places.

Progression in Multiplication and Division

MULTIPLICATION	DIVISION
Early experiences Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.	Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.
Repeated addition (repeated aggregation) 3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3 Children learn that repeated addition can be shown on a number line. Children learn that repeated addition can be shown on a bead string. Children also learn to partition totals into equal trains using Cuisenaire Rods. 5 x 3 = 15	**Sharing equally** 6 sweets get shared between 2 people. How many sweets do they each get? **Grouping or repeated subtraction** There are 6 sweets. How many people can have 2 sweets each?
Scaling This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as: 1 + 1 + 1 = 3 → scaled up by 3 → 5 + 5 + 5 = 15 For example, find a ribbon that is 4 times as long as the blue ribbon. 5 cm 20 cm We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 X 0.5 = 1.5.	**Repeated subtraction using a bead string or number line** 12 ÷ 3 = 4 The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?' Cuisenaire Rods and Numicon also help children to interpret division calculations.

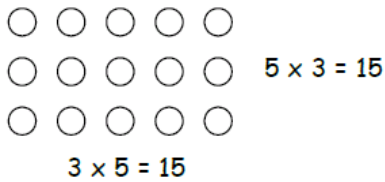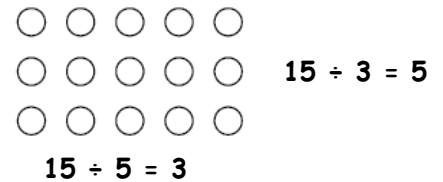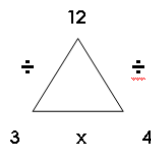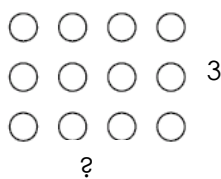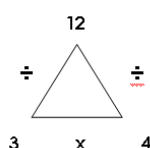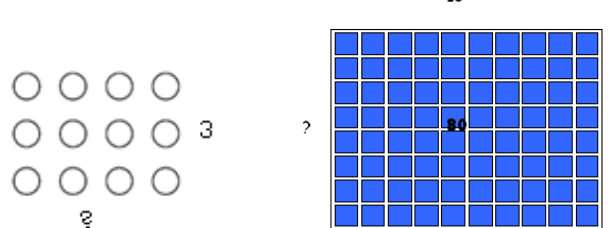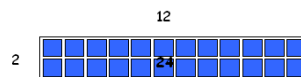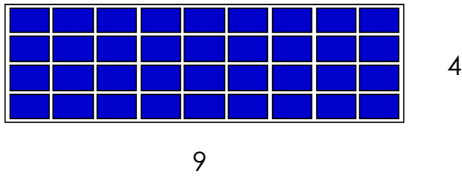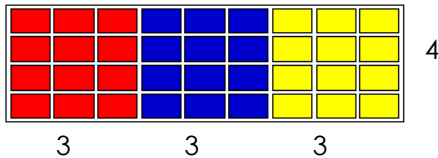|	
Commutativity Children learn that 3 x 5 has the same total as 5 x 3. This can also be shown on the number line. 3 x 5 = 15 5 x 3 = 15 	**Grouping involving remainders** Children move onto calculations involving remainders. 13 ÷ 4 = 3 r1
Arrays Children learn to model a multiplication calculation using an array. This model supports their understanding of commutativity and the development of the grid in a written method. 	**Arrays** Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method.
Inverse operation Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts. 3 x 4 = 12 4 x 3 = 12 12 ÷ 3 = 4 12 ÷ 4 = 3 Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations. · x 5 = 20　　3 x Δ = 18　　O x · = 32 This can also be supported using arrays: e.g. 3 X ? = 12 	**Inverse operation** Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts. 12 ÷ 3 = 4 12 ÷ 4 = 3 3 x 4 = 12 4 x 3 = 12 Children use symbols to represent for unknown numbers to complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations. 24 ÷ 2 = ·　　15 ÷ Δ = 3　　O ÷ 10 = 8

Partitioning

Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

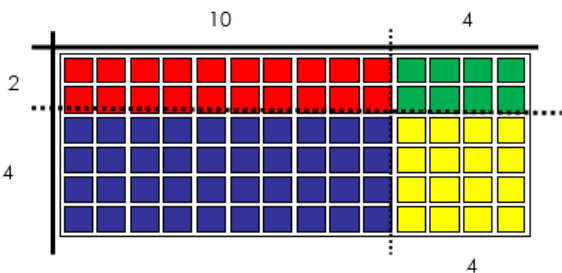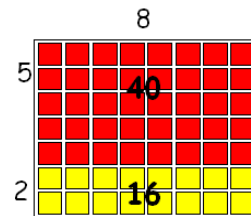

9 x 4 = 36

Children could break this down into more manageable arrays

9 x 4 = (3 x 4) + (3 x 4) + (3 x 4)
      = 12   + 12   + 12
      = 36

6 x 14 =

6 x 14 =
       = (2 x 10) + (4 x 10) + (2 x 4) + (4 x 4)
       = 20   + 40   + 8   + 16
       = 84

Partitioning

Arrays are also useful to help children visualise how to partition larger numbers into more useful arrays.

56 ÷ 8 = 7

Children could break this down into more manageable arrays, as well as using their understanding of the inverse relationship between division and multiplication.

56 ÷ 8 =

56 ÷ 8 = (40 ÷ 8) + (16 ÷ 8)
       =   5   +   2
       = 7

Arrays leading into the grid method

Children continue to use arrays and partitioning where appropriate, to prepare them for the grid method of multiplication.

Arrays can be represented as 'grids' in a shorthand version.

38 x 5 =

x	10	10	10	8
5	50	50	50	40

38 x 5 =
= (10 x 5) + (10 x 5) + (10 x 5) + (8 x 5)
= 50   + 50   + 50   + 40
= 190

Arrays leading into chunking

Children continue to use arrays and partitioning where appropriate, to prepare them for the 'chunking' method of division. Arrays are represented as 'grids' as a shorthand version.

78 ÷ 3 =

	10	10	6
3	30	30	18

$\leftarrow 78 \rightarrow$

78 ÷ 3 =
= (30 ÷ 3) + (30 ÷ 3) + (18 ÷ 3)
=   10   +   10   +   6
= 26

184 ÷ 8 =

	10	10	2	1
8	80	80	16	8

$\leftarrow 184 \rightarrow$

So 184 ÷ 8 = 23

Grid method

This written strategy is introduced for the multiplication of TU x U to begin with. It uses clear column addition methods to calculate the total.

23 x 8 =
Children will approximate first.
23 x 8 is approximately 25 x 8 = 200.

x	20	3
8	160	24

160

+   24
____
184

The vertical method- 'chunking'

78 ÷ 3 =

```
        78
-       30        (10 x 3)
        48
-       30        (10 x 3)
        18
-       18         (6 x 3)
         0
```

So 78 ÷ 3 = 10 + 10 + 6 =26

184 ÷ 8 =

```
       184
-       80        (10 x 8)
       104
-       80        (10 x 8)
        24
-       16         (2 x 8)
         8
-        8         (1 x 8)
         0
```

So 184 ÷ 8 = 23

<table>
<tr><td></td><td>

<u>Dealing with remainders</u>
Remainders should be given as integers, but children need to be able to decide what to do after division, such as rounding up or down accordingly.
e.g.
- I have 62p. How many 8p sweets can I buy?
- Apples are packed in boxes of 8. There are 86 apples. How many boxes are needed?

</td></tr>
</table>

<u>HTU x U</u>

346 x 9

Approximate first:
346 x 9 is approximately 350 x 10 = 3500

x	**300**	**40**	**6**		
9	2700	360	54		2700
				+	360
				+	54
					3114

¹ ¹

<u>TU ÷ U and HTU ÷ U- compact methods</u>
Children continue to use chunking methods to solve division calculations using increasingly more efficient methods and larger multiples of the divisor (20x, 30x).

196 ÷ 6 =

```
        196
    -   180      (30 x 6)
         16
    -    12      (2 x 6)
          4
```

196 ÷ 6 = 32 r 4

Children learn to show quotients as fractions.
196 ÷ 6 = 32 4/6   or 32 2/3

<u>TU x TU</u>

72 x 38

Approximate first:
72 x 38 is approximately 70 x 40 = 2800

x	**70**	**2**		
30	2100	60		2100
8	560	16	+	560
			+	60
			+	16
				2736

¹

<u>HTU ÷ TU</u>

977 ÷ 36 =

```
        977
    -   720      (20 x 36)
        257
    -   252      (7 x 36)
          5
```

977 ÷ 36 = 27 r 5 or 27 5/7

<u>U.t x U</u>

4.9 x 3

Approximate first:
4.9 x 3 is approximately 5 x 3 = 15

x	**4**	**0.9**		
3	12	2.7		12
			+	2.7
				14. 7

<u>U.t ÷ U</u>

87.5 ÷ 7 =

```
        87.5
    -   70.0      (10 x 7)
        17.5
    -   14.0      (2 x 7)
         3.5
    -    3.5      (0.5 x 7)
          0
```

87.5 ÷ 7 = 12.5

Children extend their use of the grid method to include:

ThHTU x U e.g. 4346 x 8 =

HTU x TU e.g. 372 x 24

U.h x U e.g. 4.92 x 3

Long division 'The bus stop method'
This brings children back to using a shorthand version of arrays, combined with skills developed through chunking.

78 ÷ 3 =

	10	10	6
3	30	30	18

$\leftarrow 78 \rightarrow$

becomes

```
    _____
3 | 78
```

```
    26
3 | 78
  - 60
    18
  - 18
     0
```

196 ÷ 6 =

```
      32 r 4
6 | 196
  - 180
     16
  -  12
      4
```
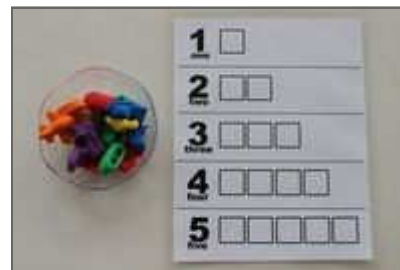
APPENDIX 1

Counting

The importance of counting can not be overstressed. Children need to be confident in the following areas:
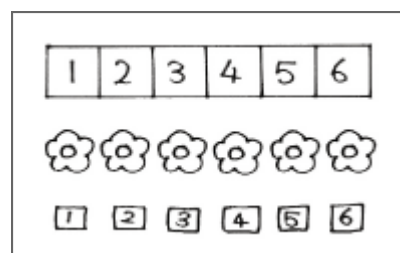
One-to-one correspondence
Children synchronise their counting and pointing, keeping track of their counting as they go, assigning one number name to one object and only counting each object once. Counting static pictures is harder, and children need to devise a system to know which they have counted as they go along.

Stable order of counting
To be able to count means knowing that the list of words used must be in a repeatable order. This principle calls for the use of a stable list that is at least as long as the number of items to be counted; if children only know the number names up to 'six', then they obviously are not able to count seven items.

Cardinal aspect of number
This is the idea that when children are counting a set of objects, the last number counted is the number of objects altogether.

Abstract principle of number
This is where children are counting things that cannot be touched or moved, such as sounds, imaginary objects or even the counting words.

Order irrelevance
Children need to understand that the order the objects are counted in doesn't matter and show that when necessary they self correct or check their counting. It does not really matter whether the counting procedure is carried out from left to right, from right to left or from somewhere else, so long as every item in the collection is counted once and only once.

Partitioning

Children need to feel confident partitioning numbers in different ways.

Greater than/ less than/ equivalent

How many more or less using direct comparison.

= as a sign of equality

Balance scales, using weighted objects, such as Cuisenaire rods, Base 10 blocks or Numicon.

Cuisenaire Roads– make trains of equal length.

Make both sides balance.

Children need to be confident in partitioning numbers in a variety of ways to ensure that they can think flexibly about numbers.

Partitioning can be explored through a wide variety of equipment and materials, all of which can represent a range of numbers and decimals.

Multilink towers– make 4

Examples of partitioning.
All complements to 1, 10, 100 etc.
Partitioning numbers in all possibilities
9
0 + 9
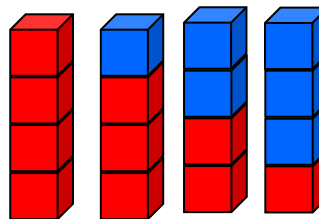1 + 8
2 + 7
3 + 6
4 + 5
Plus an infinite number of decimal combinations.

Bead strings

Cuisenaire Rods– make trains of equal length

Numicon- how many ways can you make …?

Always encourage 'Finding All Possibilities'
(*systematic working and how do you know you have them all?*)